

On the Structure and Properties of Order (K) and Degree (J) Classes of Sets

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ABSTRACT:

In this paper, we introduce a new class of sets called (k, j) -picture sets, which arise from studying collections of information subsets within a universe set. Let X be a universe set and consider k subsets $S_1, S_2, \dots, S_k$ of X . These subsets represent different portions of information contained in the universe. Our focus is on the relationships among these subsets, particularly how they share or do not share information. We assume that S_1 is a reference subset used for comparison with the remaining subsets. Among the subsets $S_2, \dots, S_k$ there exists at least one subset S_j such that it does not share information with S_1 . This condition creates a distinct structural configuration among the subsets. Based on this idea, we define the concept of a (k, j) -picture set. The definition captures situations where at least one subset remains independent of S_1 . Such configurations help in understanding patterns of separation among subsets. We further explore the structural and theoretical properties of these (k, j) -picture sets.

Keywords: picture set, picture fuzzy set, inclusion relation, soft set, (k, j) -picture set.

1. Introduction

Traditional logic assigns statements a value of either true or false, which limits its ability to represent uncertainty present in real-world situations. To address this issue, Zadeh [19] introduced fuzzy set theory, where elements belong to a set with a degree of membership ranging from 0 to 1. Since then, the theory has been expanded and refined by many researchers. Some important extensions include intuitionistic fuzzy sets by Atanassov [1], soft sets by Molodtsov [10], fuzzy soft sets [2], rough sets by Pawlak [12], vague sets and their soft variants by Wei Xu et.al [17], and intuitionistic multi-fuzzy sets by Muthuraj R et.al [11]. In addition, neutrosophic sets introduced by Smarandache [15] provide a highly general framework for analyzing and managing uncertainty.

Although this list is not exhaustive and these concepts have many variations depending on theoretical and practical needs, one notable development is the introduction of picture fuzzy sets (PFS) by Cuong [5] in 2014. PFS extend intuitionistic fuzzy sets by incorporating additional membership degrees, namely neutral and refusal, alongside the usual membership and non-membership values. This enhancement allows for a more effective representation of uncertainty, especially in situations where individuals may express multiple viewpoints such as “yes,” “no,” “abstain,” or “refusal.”

A practical example of this can be seen in general elections, where voters may support a candidate (“yes”), oppose them (“no”), abstain from voting, or refuse to choose any of the available options. Due to their flexibility,

picture fuzzy sets have been increasingly applied in decision-making problems, including areas such as material selection, mineral identification, and algebraic studies such as interval-valued picture fuzzy ideals [16].

As we can see, fuzzy sets and their extensions serve as powerful tools for handling uncertainty and vagueness. Beyond practical applications, they are also important in theoretical studies, for instance in the development of fuzzy topological spaces and their generalizations, such as supra-topologies, infra-topologies, and minimal structures.

2. Preliminaries

Let us start from some basic notions. We recall the general idea of fuzzy and intuitionistic fuzzy sets. Then they will be compared with picture fuzzy and picture sets. Moreover, we mention intuitionistic sets too.

First, let us recall the intuition that has been already mentioned. A fuzzy set (FS) is a set in which every element has a degree of membership of belonging in it. This can be formulated in a strict mathematical manner, as it is in the definition below.

Definition 2.1. [19] Let X be a non-empty universal set. A fuzzy set M on X is an object of the form: $M = \{(x, \mu_M(x)); x \in X\}$ where $\mu_M(x): X \rightarrow [0, 1]$ is a function that assigns a real number from the interval $[0, 1]$ to each element of X . The value $\mu_M(x)$ shows the grade of membership of x in M .

Definition 2.2. [1] An intuitionistic fuzzy set (IFS) M on a non-empty set X is an object of the form $M = \{(x, \mu_M(x), \nu_M(x)); x \in X\}$ where $\mu_M(x) \in [0, 1]$ is called the degree of membership of x in M , $\nu_M(x) \in [0, 1]$ is called the degree of non-membership of x in M and we assume that $\mu_M(x)$ and $\nu_M(x)$ satisfy the following condition $\forall x \in X (\mu_M(x) + \nu_M(x) \leq 1)$

Intuitionistic fuzzy sets are constantly studied in the context of topologies. For example, various subclasses of weakly open sets are

analyzed [6] for one of the most recent results. Intuitionistic sets are their crisp version.

Definition 2.3. [3, 4, 9], Let X be a non-empty universal set. We say that $M \subseteq X$ is an intuitionistic set (IS) if M is of the form $M = \langle X, M_1, M_2 \rangle$, where M_1 and M_2 are the subsets of X such that $M_1 \cap M_2 = \emptyset$. The sets M_1 and M_2 are interpreted as members and non-members of M respectively.

Definition 2.4. [3], Let X be a non-empty set, $M = \langle X, M_1, M_2 \rangle$ and $N = \langle X, N_1, N_2 \rangle$ be two IS's on X . Let J be a non-empty index set and $\{M_i : i \in J\}$ be an arbitrary family of IS's on X , where each of its elements is of the form $M_i = \langle X, M_i^{(1)}, M_i^{(2)} \rangle$ for any $i \in J$.

Definition 2.5. [5, 8] Let X be a non-empty universe. A picture fuzzy set (PFS) M on a non-empty universe X is an object of the form $M = \{(x, \mu_M(x), \eta_M(x), \nu_M(x)); x \in X\}$ where $\mu_M(x) \in [0, 1]$ is called the grade of positive membership of x in M , $\eta_M(x) \in [0, 1]$ is called the degree of neutral membership of x in M , $\nu_M(x) \in [0, 1]$ is called the degree of negative membership of $x \in M$. We assume that $\mu_M(x) + \eta_M(x) + \nu_M(x) \leq 1$ for any $x \in X$. Then $1 - (\mu_M(x) + \eta_M(x) + \nu_M(x))$ is called the degree of refusal membership of x in M .

Definition 2.6. [7, 18] Assume that X is a non-empty universe. A picture set on X is an object of the form $M = \langle X, M_1, M_2, M_3 \rangle$ where M_1 , M_2 and M_3 are the subsets of X such that $M_1 \cap M_3 = \emptyset$. The sets M_1 , M_2 , M_3 and $(M_1 \cup M_2 \cup M_3)^c$ are the sets of members, neutral members, non-members and refusal members of M respectively.

Note that in neutrosophic crisp sets we have stronger assumptions that $M_1 \cap M_2 = \emptyset$, $M_1 \cap M_3 = \emptyset$ and $M_2 \cap M_3 = \emptyset$ [13, 14]. Hence, each neutrosophic crisp set can be considered as a picture set but the converse of this statement is not true.

3. (k, j)-Picture sets

Definition 3.1: Let X be a non empty universe. Let j, k be any two positive integers such that $2 \leq j \leq k$. An object M is a picture set of order k and degree j if there is a k -tuple $\lambda_M = (M_1, M_2, M_3, \dots, M_k)$ of subsets of X such that $M_1 \cap M_j = \emptyset$. The picture set M is identified with notation

$$M = \langle X: \lambda_M \rangle. \quad (3.1)$$

For each (k, j) -picture set M , a suitable function $P_M: \{1, 2, 3, \dots, k\} \rightarrow 2^X$ may be defined as $P_M(1) = M_1, P_M(2) = M_2, P_M(3) = M_3, \dots, P_M(k) = M_k$ and the corresponding picture set will be written as $M = \langle X: (M_1, M_2, M_3, \dots, M_k) \rangle = \langle X: \lambda_M \rangle = \langle X: P_M \rangle$. (3.2)

Depending upon the nature of the definitions, results and propositions which are to be proposed, the appropriate notation of a (k, j) -picture set in (3.2) will be taken for the discussion and analysis of (k, j) -picture sets.

Since the choice of a desirable picture set of desirable order and desirable degree depends upon the selection of a set valued function $f: \{1, 2, 3, 4, \dots, k\} \rightarrow 2^X$, a (k, j) -picture set may also be taken as a soft set over X with $\{1, 2, 3, 4, \dots, k\}$ as a parameter space.

Example 3.2: Suppose the children are asked to put 10 balls in each of five boxes that are given to each child. The balls should be collected from the heap of balls available in a fixed room, subject to the following conditions.

- i. The first box whose name is specified with subscript 1, should be filled with white balls only.
- ii. The names of remaining boxes are specified with subscripts 2, 3, 4, 5.
- iii. For example, if B is the name of a child then the associated boxes are B_1, B_2, B_3, B_4, B_5 .

Suppose there are five children A, B, P, Q and R in the game. Then the boxes $(A_1, A_2, A_3, A_4, A_5); (B_1, B_2, B_3, B_4, B_5); (P_1, P_2, P_3, P_4, P_5); (Q_1, Q_2, Q_3, Q_4, Q_5)$ and $(R_1, R_2, R_3, R_4, R_5)$ are filled with balls by A, B, P, Q and R respectively. We assume that the boxes A_1, B_1, P_1, Q_1 and R_1 may be filled with white balls only. If a ball other than white is found in the first box of a child, then the child is eliminated from the game. Let X denote the collection of all balls available in the room. Suppose no white ball is found in the boxes A_2, B_5, P_4, R_3 and a white ball is found in the boxes Q_2, Q_3, Q_4, Q_5 . Then

(i) $A = \langle X: (A_1, A_2, A_3, A_4, A_5) \rangle$ is a picture set of order 5 and degree 2. That is

$A = \langle X: \lambda_A \rangle = \langle X: P_A \rangle$ is a $(5, 2)$ -picture set on X .

(ii) $B = \langle X: (B_1, B_2, B_3, B_4, B_5) \rangle$ is a picture set of order 5 and degree 5. That is

$B = \langle X: \lambda_B \rangle = \langle X: P_B \rangle$ is a $(5, 5)$ -picture set on X .

(iii) $P = \langle X: (P_1, P_2, P_3, P_4, P_5) \rangle$ is a picture set of order 5 and degree 4. That is

$P = \langle X: \lambda_P \rangle = \langle X: P_P \rangle$ is a $(5, 4)$ -picture set on X .

(iv) $R = \langle X: (R_1, R_2, R_3, R_4, R_5) \rangle$ is a picture set of order 5 and degree 3. That is

$R = \langle X: \lambda_R \rangle = \langle X: P_R \rangle$ is a $(5, 3)$ -picture set on X .

(v) The child Q is eliminated from the game as $Q_1 \cap Q_2 \neq \emptyset, Q_1 \cap Q_3 \neq \emptyset, Q_1 \cap Q_4 \neq \emptyset$ and

$$Q_1 \cap Q_5 \neq \emptyset.$$

Let $PS[X: (k, j)]$ denote the collection of all (k, j) -picture sets on X . It is identified with a subset of the Cartesian product of the k -copies of 2^X . Every (k, j) -picture set $M = \langle X: (M_1, M_2, M_3, \dots, M_k) \rangle$ induces two soft sets with set valued functions $Q_{(M, 1)}: \{1\} \rightarrow 2^X$ and $Q_{(M, j)}: \{2, 3, \dots, k\} \rightarrow 2^X$ defined as $Q_{(M, 1)}(1) = M_1,$

$Q_{(M,i)}(i) = M_i$ for $i=2,3,4,\dots,k$ with $Q_{(M,1)}(1) \cap Q_{(M,i)}(i) = \emptyset$.

Proposition 3.3: Let $M = \langle X: (M_1, M_2, M_3, \dots, M_k) \rangle$ be a (k,j) -picture sets on X . The following are equivalent

(i) $M = \langle X: \lambda_M \rangle$ is a (k,j) -picture set on X

(ii) $M \in PS[X:(k,j)]$

(iii) λ_M is a (k,j) -tuple of subsets of X .

(iv) M is associated with a set function $P_M: \{1,2,3,\dots,k\} \rightarrow 2^X$, with $P_M(1) \cap P_M(j) = \emptyset$.

(v) M is associated with a pair $(Q_{(M,1)}, Q_{(M,i)})$ of soft sets on X with $(\{1\}, [2,k] \cap \mathbb{Z})$ as the pair of parameter spaces such that $Q_{(M,1)}(1) \cap Q_{(M,i)}(i) = \emptyset$.

Proposition 3.4: If X is a finite set of universe with n members then

$|PS[X:(k,j)]| = \sum_{A \subseteq X} 2^{[(k-1)n - |A|]}$ where the summation is taken over all subsets of X .

Corollary 3.5: If X is a finite set of universe with n members then

$|PS[X:(k,j)]| = 3^n \times 2^{(k-2)n}$ where the summation is taken over $r \in \{0,1,2,\dots,n\}$.

Let $M = \langle X: (M_1, M_2, M_3, \dots, M_k) \rangle$ and $N = \langle X: (N_1, N_2, N_3, \dots, N_k) \rangle$ be any two (k,j) -picture sets on X . Then $M, N \in PS[X:(k,j)]$. Here the k -tuple $(M_1, M_2, M_3, \dots, M_k)$ is also known as a (k,j) -tuple on X .

Definition 3.6: Let $M, N \in PS[X:(k,j)]$. M is a (k,j) -picture subset of N if $M_1 \subseteq N_1, M_2 \supseteq N_2, M_3 \supseteq N_3, \dots, M_k \supseteq N_k$. If M is a (k,j) -picture subset of N , symbolically it is written as

$M^{(k,i)} \subseteq_{ps} N$ and $(k,i) \subseteq_{ps}$ is read as “is a (k,j) -picture subset of”.

If $M^{(k,i)} \subseteq_{ps} N$ we also say that M is (k,j) -picture set included in N .

It is customary to write $M = N$ if $M_1 = N_1, M_2 = N_2, M_3 = N_3, \dots, M_k = N_k$.

Definition 3.7: Let the (k,j) -tuples $(X_1, X_2, X_3, \dots, X_k)$ and $(Y_1, Y_2, Y_3, \dots, Y_k)$ are subsets of X and Y respectively. A (k,j) -tuple $(X_1, X_2, X_3, \dots, X_k)$ is said to (k,j) -dominate $(Y_1, Y_2, Y_3, \dots, Y_k)$ if $X_1 \subseteq Y_1$ and $X_i \supseteq Y_i$ for every $i=2,3,\dots,k$ in which case we say that $(X_1, X_2, X_3, \dots, X_k)$ is a (k,j) -domination of $(Y_1, Y_2, Y_3, \dots, Y_k)$.

Definition 3.8 : A set function $f: [1,k] \cap \mathbb{Z} \rightarrow 2^X$ is said to (k,j) -dominate set function $g: [1,k] \cap \mathbb{Z} \rightarrow 2^X$

if $f(1) \subseteq g(1)$ and $f(i) \supseteq g(i)$ for every $i=2,3,\dots,k$ in which case we write

$(k,i) [f \triangleright g]$ (read this as f is a (k,j) -domination of g).

Theorem 3.9: Let $M = \langle X: (M_1, M_2, M_3, \dots, M_k) \rangle$ and $N = \langle X: (N_1, N_2, N_3, \dots, N_k) \rangle$ be any two (k,j) -picture sets on X . The following are equivalent

(i) M is a (k,j) -picture subset of N .

(ii) $(M_1, M_2, M_3, \dots, M_k)$ is a (k,j) -domination of $(N_1, N_2, N_3, \dots, N_k)$.

(iii) λ_M is a (k,j) -domination of λ_N

(iv) P_M is a (k,j) -domination of P_N

(v) $Q_{(M,1)}$ is a soft subset of $Q_{(N,1)}$ and $Q_{(N,i)}$ is a soft subset of $Q_{(M,i)}$.

Proposition 3.10: Let X be a finite set of universe with n objects, $M, N \in PS[X:(k,j)]$, $M = \langle X: (M_1, M_2, M_3, \dots, M_k) \rangle$ with $A = M_1$, $|M_i| = m(i)$ and $N = \langle X: (N_1, N_2, N_3, \dots, N_k) \rangle$ with $B = N_1$, $|N_i| = n(i)$ for $i=1,2,3,\dots,k$. If $PS_N[X:(k,j)]$ is the set of all (k,j) -picture subsets of N then the number of (k,j) -picture subsets of N is given by

$|PS_N[X:(k,j)]| = \sum_{A \subseteq B} 2^{(n-m(1)-n(j))} \times \prod \{2^{n-n(i)} : i \neq 1, i \neq j\}$, $m(1) = |A|$ where the product is taken over $i \neq j, i \neq 1$ and the summation is taken over the subsets of N_1 subject to the condition that $M_1 \cap M_j = \emptyset$.

Corollary 3.11: If X is a finite set of universe with n members then $|PS_N[X:(k,j)]| = 3^{n(1)} \times 2^{(k-1)n - |N|}$ where $|N| = |N_1| + |N_2| + |N_3| + \dots + |N_k|$ and $n(1) = |N_1|$.

Definition 3.12 The (k, j) -empty picture set and (k, j) -absolute picture set are respectively defined as $[\emptyset]_{(k,j)} = \langle X: P_{\emptyset} \rangle$ and $[X]_{(k,j)} = \langle X: P_X \rangle$.

Clearly $[\emptyset]_{(3,3)} = \langle X: (\emptyset, X, X) \rangle$ and $\langle X: P_X \rangle = \langle X: (X, \emptyset, \emptyset) \rangle$ which are due to Durga et al. and $[\emptyset]_{(2,2)} = \langle X: (\emptyset, X) \rangle$ and $\langle X: P_X \rangle = \langle X: (X, \emptyset) \rangle$

Definition 3.13: If A is a proper subset of X then $[A]_{(k,j)} = \langle X: P_{(A,1)} \rangle$,

Remark 3.14: $[A]_{(k,j)}$ is a (k, j) -picture set on X determined by a proper subset A of X .

Definition 3.15 : If $x \in X$ then $[x]_{(k,j)} = [\{x\}]_{(k,j)} = \langle X: P_{(x,1)} \rangle$

Proposition 3.16 : $[\emptyset]_{(k,j)}^{(k,i)} \subseteq_{ps} [A]_{(k,j)}^{(k,i)} \subseteq_{ps} [X]_{(k,j)}$ for every proper subset A of X .

4. Conclusion

In this paper we introduced a new type of set called (k, j) -picture sets. These sets can be considered as a generalization of picture sets. They also have a close relationship with intuitionistic sets and picture sets. The proposed sets extend the existing concepts and provide a broader framework for analysis. In addition, the authors discuss the connections between these sets and previously defined set models. The main properties of the introduced sets are examined in this study. Furthermore, appropriate counter examples are provided to clarify and justify the proposed concepts.

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Author Declaration

We hereby declare that the manuscript submitted to the International Journal of Special Education is an original work and has not been published previously, nor is it currently under consideration for publication elsewhere

We further declare that there are **no conflicts of interest** associated with this work. All authors have contributed to the preparation of the manuscript and have approved the final version submitted for publication.