Didactic-Mathematical Knowledge and Instructional Design in Blended Mathematics Teaching Under Conditions of Unequal Digital Access: A Multiple Case Study in Central America

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ABSTRACT:
This study examined how secondary school classrooms, within a higher education training context mathematics teachers participating in the Diploma in Mathematics Education Applied to Secondary Education (EMAS), conceived as a blended professional development environment, mobilized didactic-mathematical knowledge and competencies when designing, implementing, assessing, and redesigning instruction under unequal conditions of digital access and participation. Grounded in the Didactic-Mathematical Knowledge and Competencies model and the criteria of didactic suitability and complemented by the TPACK-XK framework and the Theory of Instrumental Genesis, the study adopted a qualitative, interpretive multiple-case design. Three teachers, identified as A, B, and G, were analyzed through their didactic sequences, classroom implementation videos and transcripts, classroom evidence, teacher reflections, and redesign proposals. Findings revealed a shared professional basis across the three cases, but also important differences in the enactment of blended instruction. Case A showed more restricted blended enactment and limited integration of the virtual component; Case B displayed a more balanced profile, with digital resources serving as bridging elements between virtual and face-to-face work; and Case G presented the strongest evidence of blended enactment, with

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digital tools fully integrated into participation, feedback, and instructional adjustment. The results suggest that differences in digital access and participation shape not only the visibility of blended learning practices, but also the ways teachers sustain instructional continuity across modalities. The study highlights the importance of designing blended teacher education environments that support coherent pedagogical pathways under unequal participation conditions.

Keywords: blended learning; teacher professional development; mathematics teacher education; digital participation; instructional design; multiple case study.

1. Introduction

The education of mathematics teachers continues to face an underlying problem: it is not enough to know the content that is taught (Schoenfeld & Kilpatrick, 2008; Godino et al., 2017). It also matters how that content is organized, how it is put into play in the classroom and how the teacher later reviews what happened during the class. At that point, the design, implementation, assessment and redesign of mathematical tasks cease to be isolated moments and become part of the teacher's professional work. This question is especially relevant when seeking to understand the knowledge and skills that teachers mobilize in their practice (Godino et al., 2017; Schoenfeld & Kilpatrick, 2008). This issue becomes particularly relevant in blended learning contexts, where instructional decisions must be distributed across face-to-face and virtual environments.

In Panama, research on teacher training in mathematics education has shown important advances in the study of the didactic suitability and professional competencies of teachers (Chacón-Rivadeneira et al., 2024; Morales-Maure et al., 2025; Calle et al., 2025). However, there is still a need to better understand how teachers mobilize didactic knowledge within blended teacher education contexts, particularly in relation to the articulation of face-to-face and virtual components. Part of the existing background has been concentrated at other educational levels or in training contexts where unequal digital participation has not occupied a central place in the analysis (Chacón-Rivadeneira et al., 2024; Mishra, 2019). This absence weighs heavily, above all, in the case of secondary school classrooms, within a higher education training context, where working conditions can vary significantly between schools and where technological mediation is not distributed homogeneously, making it difficult to ensure instructional continuity across modalities (Mishra, 2019; Mishra & Koehler, 2006).

In this structure, the question that guides the study is the following: How do secondary school teachers participating in a higher education professional development program participating in a higher education professional development program participating in a higher education professional development program participating in a higher education professional development program participating in a blended professional development environment mobilize didactic-mathematical knowledge and competencies when designing, implementing, assessing, and redesigning mathematics instruction under unequal conditions of digital access and participation? Based on this question, the aim of this article is to analyze how secondary school teachers participating in a higher education professional development program participating in a higher education professional development program participating in a higher education professional development program participating in a higher education professional development program participating in EMAS, conceived as a blended professional development environment,

mobilize didactic-mathematical knowledge and competencies when designing, implementing, assessing, and redesigning mathematics instruction across face-to-face and virtual learning conditions.

To do this, a multiple case study is adopted with three teachers identified as A, B and G. The analysis is mainly based on the model of Didactic-Mathematical Knowledge and Competences of the teaching staff and on the criteria of didactic suitability, incorporating in a complementary way the TPACK-XK and the Theory of Instrumental Genesis (Godino et al., 2017; Breda et al., 2018; Breda et al., 2021; Mishra & Koehler, 2006; Mishra, 2019; Trouche, 2004). From there, the article examines the relationship between didactic sequence, implementation, classroom evidence, teacher assessment or reflection, and redesign, with attention to the concrete conditions of mediation in which the practice is developed. This orientation makes it possible to examine not only teachers' didactic work, but also how blended learning conditions shape their instructional decisions, participation structures, and redesign processes.

2. Theoretical framework

The study is based on the model of didactic-mathematical knowledge and competence of the teaching staff, on the criteria of didactic suitability and, in a complementary way, on the TPACK-XK and the Theory of Instrumental Genesis.

2.1. Didactic-mathematical knowledge and skills of teachers

Godino et al. (2017) develop the Didactic-Mathematical Knowledge and Competencies model of the mathematics teacher (CCDM) to characterize the professional teaching knowledge in relation to the teaching of mathematics. In this model, two fundamental competences are distinguished: mathematical competence, linked to knowledge of the content, and the competence of analysis and didactic intervention, related to the design, application and assessment of learning sequences, as well as to the formulation of proposals for improvement. In addition, the latter includes sub-competences linked to the analysis of mathematical activity, the management of interaction, the recognition of norms and meta-norms and the assessment of the didactic suitability of instructional processes (Breda et al., 2017). In blended learning contexts, these competencies must also be enacted across distributed instructional environments, where planning, mediation, and reflection extend beyond the physical classroom.

Within the CCDM, teachers' knowledge is organized into three dimensions: mathematical, didactical, and metadidactical (Godino et al., 2017). The mathematical dimension refers to the knowledge necessary to solve tasks at the level at which it is taught. The didactical dimension brings together the knowledge linked to the planning and implementation of teaching and is structured in six facets: epistemic, cognitive, affective, interactional, mediational and ecological. The metadidactical dimension, on the other hand, encompasses the knowledge necessary to reflect on one's own practice, evaluate instructional processes, and carry out redesigns aimed at improvement (Schön, 1983; Schoenfeld & Kilpatrick, 2008).

2.2. Criteria of didactic suitability as a tool for analysing teaching practice and productions

Within the CCDM model, the notion of didactic suitability allows us to assess the processes of mathematical teaching and learning. Godino et al. (2017) defines it as the degree to which an instructional process presents adequate characteristics to articulate the personal meanings constructed by students with the intended institutional meanings, considering the circumstances and resources of the environment. This notion is specified in six criteria: epistemic, cognitive, interactional, mediational, affective and ecological. In turn, these criteria are broken down into components and indicators that allow assessments and self-assessments to be made about the design and development of mathematical instruction processes (Breda, Font, & Pino-Fan, 2018).

The criteria of didactic suitability are tools for analyzing tasks, evaluations and redesigns

prepared by teachers, aimed at improvement. In recent works, Calle et al. (2024) demonstrate that CIDs provide a useful basis for evaluating proposals for improvement in teaching, while Sol et al. (2025) highlight their role in structuring practical arguments and justifying didactic modifications.

Studies indicate that the planning, implementation and revision of teaching allow establishing links between theory and practice and favor teacher professional development (Grossman et al., 2009; Kieran et al., 2012). However, this reflection does not strengthen on its own. Perrenoud (2004) warns that tools and conceptual frameworks are required to direct teachers' attention to relevant aspects of their practice.

From this perspective, teacher reflection involves evaluating decisions, interpreting their effects, and formulating adjustments for future implementations (Schön, 1983; Schoenfeld & Kilpatrick, 2008). In the field of mathematics education, this logic is clearly expressed when teachers design tasks, implement them, evaluate them in the light of evidence, and develop a redesign. In this direction, Sol et al. (2025) shows that reflection on didactic proposals acquires greater density when it is supported by explicit assessment criteria, while Fernandes da Silva et al. (2025) emphasize that the planning, execution and evaluation of training practices contributes to the development of teacher autonomy and decision-making. In this sense, the criteria of didactic suitability not only support the analysis of mathematics teaching but also make it possible to examine the coherence between face-to-face and virtual components, the use of resources, and patterns of participation in blended learning environments. Table 1 presents a synthesis of the six criteria considered in this study.

Table 1.

Criteria of didactic suitability

Criteria	Description
Epistemic suitability	It refers to the quality of the mathematics taught, that is, to the relevance, correctness and richness of the mathematical content.
Cognitive Suitability	It refers to the adequacy of the proposed learning to the area of potential development of the students and their previous knowledge.
Interactional Suitability	It is linked to the modes of interaction that favor the identification and resolution of conflicts of meaning during the teaching and learning process.
Mediational suitability	It is related to the availability, adequacy and use of material, technological and temporal resources in the development of teaching.
Affective suitability	It refers to the degree of involvement, interest, motivation and disposition of the students towards mathematical activity
Ecological Suitability	It refers to the adequacy of the teaching process to the curriculum, to the educational project of the school and to the social and institutional environment.

Note: Authors' elaboration based on Breda, Font and Pino-Fan (2018)

2.3. Blended learning, didactic mediation, and unequal conditions of digital participation in secondary school classrooms, within a higher education training context

The TPACK framework emerged as an expansion of pedagogical content knowledge by explicitly incorporating technology as a core component of teachers' professional knowledge (Mishra & Koehler, 2006). From this perspective, effective teaching with technology is not limited to technical proficiency, but involves the ability to integrate technological, pedagogical, and disciplinary knowledge in ways that are

instructionally meaningful. In this study, blended learning is understood as a pedagogical approach that goes beyond the mere use of digital tools and instead emphasizes the intentional integration of face-to-face and virtual activities within a coherent instructional design. Mishra (2019) later extended this model by incorporating contextual knowledge through the TPACK-XK framework, emphasizing that technology integration must be understood within the institutional, cultural, and material conditions in which teaching occurs. This perspective is especially relevant in secondary education, where differences in access to devices, connectivity, classroom infrastructure, and institutional support shape non-equivalent opportunities for teaching and learning.

In mathematics education, this perspective can be analytically grounded through the criteria of didactic suitability, particularly the mediational and ecological dimensions (Godino et al., 2017; Breda et al., 2018). These criteria make it possible to examine how face-to-face and virtual integration is structured within a blended learning design, highlighting that the key issue is not only the availability of resources, but how teaching is organized as distributed instruction across different spaces. The mediational criterion focuses on the adequacy and use of resources within mathematical activity, while the ecological criterion situates these decisions within broader institutional conditions, including forms of digital participation that shape opportunities for engagement and learning.

A complementary perspective is provided by the Theory of Instrumental Genesis (Trouche, 2004), which distinguishes between an artifact and its transformation into an instrument through use. In this study, this process is understood within a blended instructional sequence, where the meaning of an artifact emerges from its role in supporting continuity across modalities. Calculators, videos, digital platforms, or interactive applications acquire didactic value only when they are integrated into a blended enactment that connects in-person and virtual activities, organizes participation, and supports students' engagement with mathematical content.

Recent research has emphasized the importance of examining the relationship between didactic knowledge and teaching practices in context-sensitive ways (Chacón-Rivadeneira et al., 2024). In line with this perspective, the present study focuses on how teachers mobilize didactic-mathematical knowledge and competencies within processes of blended enactment, paying particular attention to how digital participation shapes instructional decisions. Specifically, it examines how teachers organize distributed instruction, sustain continuity across modalities, and articulate face-to-face and virtual integration under blended conditions with unequal digital participation, highlighting the role of context in shaping teaching and learning opportunities.

These perspectives provide the conceptual basis for analyzing technological mediation as part of situated instructional practice. Rather than treating technology as an isolated variable, this study examines how it is embedded in the broader processes of task design, classroom enactment, reflection, and redesign. This orientation justifies the methodological decision to adopt a multiple-case study design, through which the mobilization of didactic-mathematical knowledge and competencies can be explored across secondary school classrooms, within a higher education training context characterized by unequal access to and use of technological resources.

3. Methodology

This section outlines methodological decisions guiding the study, focusing on how teachers' professional practices are examined within a blended professional development context.

3.1. Study Approach and Design

The study is situated within a qualitative, interpretative approach. It seeks to understand how secondary school teachers participating in a higher education professional development program participating in a higher education professional development program participating in a higher education professional development program participating in a higher education professional development program mobilize

didactic-mathematical knowledge and skills in a blended professional development setting, particularly as they design, implement, assess, and reformulate mathematical tasks across face-to-face and virtual components. In this sense, the analysis pays special attention to how teachers transfer, adapt, and enact what is developed in one modality into another, as part of an integrated instructional process. For this reason, a multiple case study design was chosen, responding to the need to carefully examine a common training process in classroom situations that are not equivalent to each other, while reconstructing trajectories of design and redesign across modalities.

This type of design facilitates the management of a wide and diverse set of evidence produced in the context of the diploma course, allowing the relationship between what is planned, what is implemented, and the subsequent assessment by each teacher to be addressed within a blended framework. Rather than comparing isolated performances, the aim is to reconstruct professional trajectories, paying attention to how instructional decisions evolve between face-to-face and virtual spaces, and to recognize regularities, differences, and tensions in these processes of enactment and redesign across modalities.

3.2. Training context: EMAS

The research was carried out within the framework of the Diploma in Mathematics Education Applied to Secondary Education (EMAS), conceived as a continuous professional development program for secondary school mathematics teachers. In this study, EMAS is understood as a blended professional development environment, in which face-to-face and virtual components are pedagogically integrated. EMAS combined face-to-face sessions with structured virtual activities through an online platform, creating a blended professional development environment in which participants engaged with content, instructional planning, reflection, and evidence-based redesign. Within this framework, participants worked on mathematical content, analyzed teaching decisions, and developed didactic sequences connected to their classroom contexts. The face-to-face sessions were oriented toward the collective exploration of mathematical tasks, the discussion of didactic criteria, and the analysis of teaching practices through guided activities and group interactions. These sessions created opportunities for shared reflection, peer discussion, and direct support from facilitators. In parallel, the virtual platform hosted a set of structured activities that extended and complemented in-person work. These included access to instructional materials such as readings, videos, and guidelines; asynchronous tasks such as the design of didactic sequences and written reflections; and spaces for interaction, including forums where participants shared ideas, discussed challenges, and provided peer feedback. The platform also functioned as a repository of evidence, where teachers uploaded lesson plans, classroom records, student work, and reflective analyses, receiving feedback from both peers and instructors. In this way, the program articulated synchronous and asynchronous work, ensuring continuity between modalities.

For this article, the final module of the diploma course is especially relevant, as it required participants to engage in a complete cycle of professional practice within this blended environment. Each teacher designed a sequence of tasks, implemented it in their classroom, documented the process through collected evidence, reflected on the experience using the criteria of didactic suitability, and proposed a redesign. These activities were distributed across face-to-face and virtual components, requiring teachers to connect planning, enactment, and reflection across modalities. Therefore, the focus of the study is not limited to written planning, but rather on how teachers develop, enact, document, and revise instructional sequences within a blended ecosystem that supports ongoing reflection and improvement.

3.3. Selection of cases A, B and G

The cases were also selected because they allowed the examination of how a shared blended professional development framework was enacted under different school and digital participation conditions. Three teachers identified as A, B and G were chosen, whose productions were complete and allowed the reconstruction of the process worked on in the final module of the EMAS. In all three cases, there was a final didactic sequence, video record of the implementation, transcription, classroom evidence, teacher assessment or reflection, and redesign proposal. In addition to this criterion of documentary completeness, the choice responded to an analytical reason. The three cases made it possible to examine the work of teachers in school contexts with different conditions of access and use of technological resources. This diversity was not treated as a methodological difficulty. On the contrary, it was part of the research problem and the comparative sense of the study. Table 2 presents a synthetic characterization of the selected cases, considering the grade, the mathematical topic, the resources identified, the role of the resource in the class, and the availability of sources for analysis.

3.4. Analysis of sequences, video, classroom evidence and teacher assessment/reflection

The data were integrated into four types of evidence. First, the didactic sequence developed by each teacher. Second, the implementation video and its transcription. Third, the classroom evidence generated during the development of the class. Finally, the assessment or reflection of the teacher accompanied by the redesign proposal. These sources were read in an articulated way. It was not of interest to describe each document separately, but rather to relate them in order to reconstruct the instructional process as a whole. The sequence made it possible to recognize what was planned by the teacher; the video and transcript showed how this proposal was effectively enacted in the classroom; the classroom evidence provided traces of how the experience unfolded; and the final reflection allowed us to observe which decisions were reviewed, sustained, or modified.

This cross-analysis was necessary to infer knowledge and competencies from documented practice, and not only from the declarative plane. Additionally, it made it possible to examine how teachers articulated face-to-face and virtual components, how they adapted their instructional decisions to conditions of unequal participation, and how the blended environment influenced processes of redesign and improvement.

3.5. Analysis and inference tool

The inference of knowledge and competencies was made from an analysis tool supported by three references: the model of Didactic-Mathematical Knowledge and Competences of the teaching staff (CCDM), the criteria of didactic suitability, and, in a complementary way, the TPACK-XK model and the Theory of Instrumental Genesis. The main weight of the analysis fell on the criteria of didactic suitability and its components, since they offered a more direct way to read the practice, the teaching evaluations, and the proposals for redesign. In particular, the CCDM and the criteria of didactic suitability were used to read phenomena specific to blended learning, such as continuity across modalities, digital mediation, participation structures, and the adaptation of instructional design in blended conditions.

The other two referents fulfilled a more limited function. They were incorporated when it was necessary to interpret the blended mediation of tasks and the instructional role of digital resources within a face-to-face/virtual learning sequence. Consequently, the tool was not used here as an object of methodological construction, but as an analytical basis to examine the body of evidence of the three teachers and support the comparative inference.

Table 2. Characteristics of cases A, B, and G

Case	Level	Theme	Resource Identified	Role of the resource in the classroom	Blended learning feature(s)	Virtual–face-to-face articulation	Type of digital participation	Sources available for analysis
A	8°	Addition and subtraction of heterogeneous fractions	Board, markers	Complementary	Minimal digital enactment; predominance of in-person explanation	Weak articulation: virtual components mainly limited to documentation rather than instructional use	Low; limited to submission of materials and reflection in the platform	Didactic sequence, video for classroom evidence, assessment, teacher reflection and redesign of their class
B	7°	Pythagorean theorem	Supporting video and guide as class material; board, markers	Supplemental for student use	Use of video and guide as bridging resources within blended learning design	Moderate articulation: virtual resources (video and guide) support and extend classroom activity	Medium; students engage with digital materials that connect virtual and in-class work	Didactic sequence, video for classroom evidence, assessment, teacher reflection and redesign of their class
G	11°	Proportion, rule of three, direct and inverse proportionality	Mentimeter, supporting video, calculator; board, markers	Complementary	Stronger integration of digital tools within blended enactment	High articulation: digital tools actively integrated into classroom interaction and linked with virtual activities	High; active and interactive digital participation (e.g., real-time responses, engagement through tools)	Didactic sequence, video for classroom evidence, assessment, teacher reflection and redesign of their class

Source: Prepared by the author based on her own analysis.

3.6. Analysis procedure

The analysis was developed in three moments. At first, a comprehensive reading of the data of each case (A, B, and G) was carried out. This allowed us to recognize the relationship between the designed sequence, the implementation, the evidence produced in the classroom, and the teacher's subsequent reflection, while also attending to the role of the virtual component, the continuity between design and enactment, and the decisions conditioned by the blended learning model. In a second moment, the analysis tool was applied to the documentary set of each case, with attention to the criteria of didactic suitability and, where appropriate, to aspects related to technological mediation and the articulation of face-to-face and virtual components. In a third moment, a comparative reading was developed between the three cases, aimed at identifying coincidences, differences, and points of tension in the mobilization of knowledge and didactic-mathematical skills, again considering the influence of blended conditions on instructional decisions.

Each inference was contrasted with the rest of the available data. Thus, what appeared in the sequence was revised against what was observed in the implementation, paying attention to the continuity across modalities; what was affirmed in the teachers' reflection was confronted with

the evidence of the classroom and the virtual activities; and redesign decisions were read in relation to difficulties or adjustments recognized during the experience and shaped by the blended learning environment. This procedure helped to maintain a less superficial reading of the process followed by each teacher, explicitly considering how face-to-face and virtual components interacted in practice.

3.7. Ethical considerations and criteria of rigor

Participants were identified only by labels A, B, and G. This coding was maintained throughout the analysis and in the presentation of the results, to protect their identity and that of the schools involved. Likewise, the study was carried out based on the documentation generated within the framework of the diploma course and used for academic and research purposes. The rigor of the analysis was sustained by contrasting sources, reading the data in each case in full, and seeking coherence between planning, implementation, assessment, and redesigning.

4. Results

This section presents the findings of the multiple case study conducted with three secondary school teachers participating in a higher education professional development program participating in a higher education professional development program participating in a higher education professional development program participating in a higher education professional development program participating in the EMAS Diploma Program. The analysis examines how each teacher enacted blended instruction, how the articulation between face-to-face sessions and virtual activities was expressed in unequal ways, and how this influenced student participation, feedback, and the redesign of tasks. The cases are presented separately in order to identify both common patterns and specific differences in the ways didactic work was sustained, how digital and in-person components interacted, and how the blended learning environment shaped instructional decisions in practice.

4.1. Case A: more restricted blended enactment in a rural/regional context

Case A corresponds to an 8th grade teacher who developed a didactic sequence on fractions, emphasizing the addition and subtraction of homogeneous and heterogeneous fractions. From the point of view of content, the sequence presents a clear organization of the topic, with defined procedures, solved examples, and practice activities. This allows us to recognize consistent work in the epistemic dimension. However, the joint review of the didactic sequence, the implementation video, and the classroom evidence shows that the development of the class relied more strongly on face-to-face explanation, with the board and marker as the primary resources, and a lower integration of the virtual component into classroom decision-making.

In this case, the blended component appears less visible in classroom enactment, and instructional continuity relies more strongly on face-to-face explanation than on digitally mediated participation. The teacher consistently monitors the pace of the class, reviews previous knowledge, proposes exercises on the board, and checks students' answers before moving to more complex situations. This dynamic reveals a relevant interactional dimension, although it is centered on teacher-student exchange rather than collaborative work between peers. Cognitively, the case shows careful attention to content sequencing and review of prior operations, but also a dependence on already stabilized solution schemes.

The mediational dimension is less developed in this case. The teaching assessment and reflection recognize limitations associated with the availability of resources, which restrict the incorporation of more varied technological or material supports. This restriction does not prevent class development, but it conditions the ways content is mediated and how digital participation could occur. Nevertheless, case A should not be read from a deficit perspective: the didactic sequence includes contextualized

problems, and the teacher's reflections reveal a favorable disposition toward student engagement and ongoing revision of practice. Overall, the case illustrates a solid mobilization of didactic-mathematical knowledge in epistemic, interactional, affective, and ecological dimensions, albeit with clearly limited integration of the virtual component and restricted blended enactment.

4.2. Case B: bridging resources for blended enactment in an urban context

Case B corresponds to a 7th grade teacher who worked on the Pythagorean theorem. The didactic sequence shows a well-defined structure: retrieval of previous knowledge, introduction of the content, verification activities, and application exercises. From the beginning, the proposal combines conceptual explanation, geometric representation, and the use of digital resources. The supporting video and guide can be interpreted as bridging elements between the virtual component of the training environment and the face-to-face enactment of instruction, facilitating continuity across modalities and supporting instructional decisions in both spaces. This gives the case a more balanced epistemic and cognitive basis, as the content is not only presented correctly, but also through an organization that promotes progressive understanding and links virtual preparation with classroom activity.

In the implementation, more favorable resources and classroom conditions are observed. The teacher uses the video and guide to connect virtual tasks with face-to-face group work, organizes students in small groups, guides the construction of squares on the sides of the right triangle, and provides continuous feedback. This set of decisions strengthens the mediational dimension while reinforcing the interactional dimension. The class incorporates moments of exploration and verification that allow students to relate content to specific representations. In this sense, case B shows broader mediation than case A, with digital resources actively bridging virtual and in-person participation, although still subordinated to the general logic of the sequence rather than to intensive technology use.

The teacher's assessment and reflection confirm this reading. On the one hand, it recognizes the interest, participation, and engagement of the group. On the other hand, it identifies aspects that could be adjusted, especially in the articulation of time, work rhythm, and content consolidation across modalities. This combination of careful planning, implementation using bridging resources, and subsequent critical review makes case B particularly stable in the epistemic, cognitive, interactional, and affective dimensions. Ecological and mediational dimensions are also present, although with lower density than in case G. Overall, this is a balanced blended enactment, where the available infrastructure expands possibilities for mediation but does not replace the need for informed teaching decisions.

4.3. Case G: strongest evidence of blended enactment in upper secondary

Case G corresponds to an 11th grade teacher who worked on ratio, proportion, and rule of three. Case G presents the strongest evidence of blended enactment, as digital tools are not peripheral support but integral elements in participation, feedback, and instructional adjustment. Unlike the two previous cases, the integration of technological resources is evident from the didactic sequence itself and sustained throughout implementation. The proposal includes clear objectives, structured activities, activation of prior knowledge, and the use of tools such as Mentimeter, supporting video, and calculators. From an epistemic perspective, the content is well defined and closely articulated with the learning indicators foreseen for the level.

During implementation, technology does not act merely as an accessory resource. Mentimeter enables real-time collection of student responses, activation of participation, and adjustment of class development based on student input. The supporting video and calculator serve explicit didactic purposes, bridging virtual preparation with face-to-face enactment. In this case, the

mediational dimension reaches its strongest expression, not only due to resource availability but also because of how these tools are embedded in content work. The interactional dimension is particularly visible, with the teacher alternating questions, confirmations, collective corrections, and digitally mediated participation. Cognitively, the class follows an orderly entry into the topic through brainstorming, review of prior concepts, and problem situations that require distinguishing types of proportional relationships.

The teacher's reflection and classroom evidence show careful attention to student participation and highlight adjustments to mediation when practical difficulties arise, such as visibility or monitoring issues. In the ecological dimension, the case is robust, connecting content to applied situations while maintaining coherence with upper secondary expectations. Overall, case G demonstrates the clearest and most visible mobilization of didactic-mathematical knowledge and skills in a blended learning environment, with strong digital participation and fully integrated face-to-face and virtual resources, while preserving consistency across epistemic, cognitive, interactional, affective, mediational, and ecological dimensions.

5. Discussion

The comparative reading of cases A, B, and G shows differences in the ways didactic work was sustained and in the role that mediation played in each case. These differences do not depend solely on the content addressed. The conditions of blended enactment, digital participation conditions, and the articulation between face-to-face and virtual components also shape how each teacher designs the didactic sequence, implements it, assesses the learning process, reflects on practice, and carries out redesign.

5.1. Knowledge and skills mobilized in unequal conditions.

The comparison of cases A, B, and G allows us to observe differences in the ways didactic-mathematical knowledge and skills are mobilized. These differences do not appear only in the didactic sequence. They are also noticeable in the implementation, in the classroom evidence, in the teacher's assessment or reflection, and in the redesign. In the epistemic dimension, the three teachers adequately develop mathematical content. The difference lies in the organization of the content: in A, explicit procedures and a more linear resolution sequence predominate; in B, the content is reinforced with representations and visual verification; and in G, the sequence appears fully articulated with the objectives and with the development of activities across modalities.

In the cognitive dimension, further nuances are observed. In A, the teacher draws on prior knowledge and explains step by step, relying more on previously stabilized solution schemes. In B and G, there is more anticipation of potential difficulties and a more careful progression through the tasks. The difference does not indicate an absence of cognitive work in A, but a distinct way of sustaining it.

The most relevant difference appears in the way blended learning was enacted through instructional mediation, digital participation, and the articulation of face-to-face and virtual resources. In A, the blended component is less visible, with stronger reliance on face-to-face explanation and minimal integration of the virtual component. In B, digital resources such as video and guide serve as bridging tools between virtual preparation and classroom enactment. In G, digital tools are fully embedded in the instructional sequence, shaping participation, feedback, and the ongoing redesign of tasks. Across these cases, the visibility and effectiveness of the blended model vary according to how each teacher mobilizes resources, organizes participation, and sustains instructional continuity across modalities.

To visualize these differences comparatively, Figure 1 presents the profile of the three cases in the six dimensions considered.

Figure 1.

Comparative profile of cases A, B and G

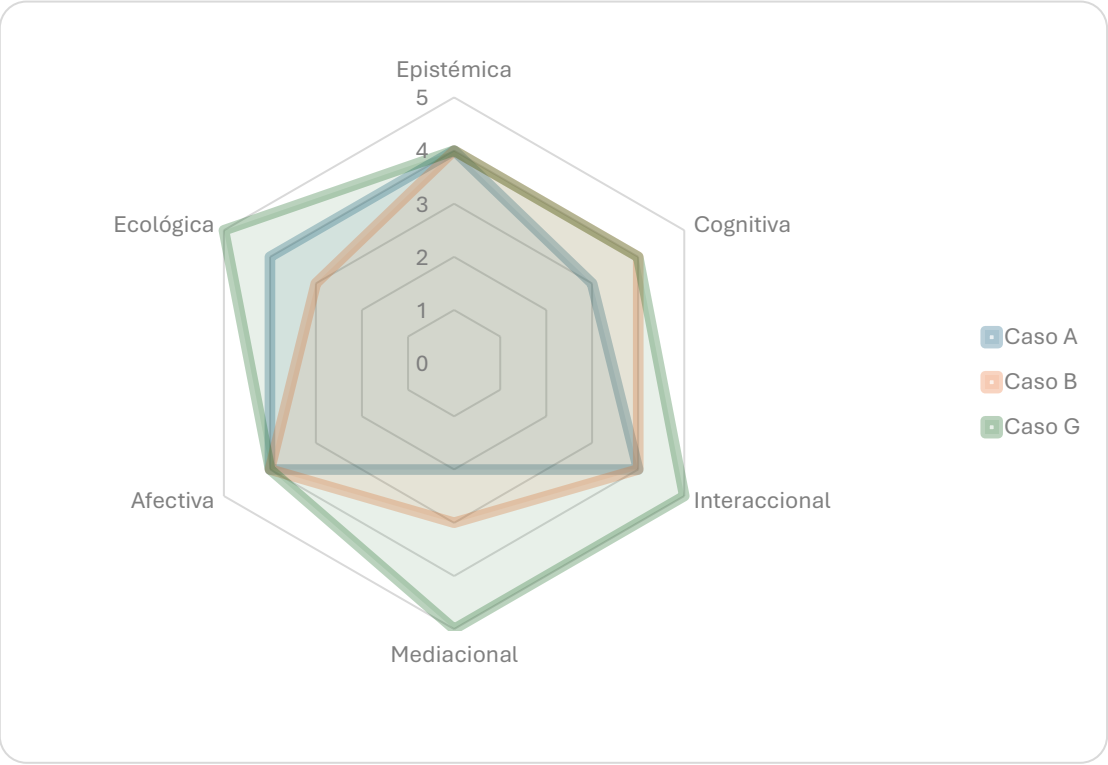

Figure 1 shows that A relies on a more restricted mediation, focused mainly on the board, marker and oral explanation. In B, mediation is extended through visual support and work material. In G, technological resources occupy a more visible place in the development of the class. The difference is not only in the presence of resources, but in the role, they play in the didactic activity.

In the interactional dimensions there are also differences. A is based more on the direct exchange between teachers and students. B combines group work and close feedback. G articulates oral participation and participation mediated by digital resources. Something similar happens in the affective dimension: in all three cases there is student involvement, although this involvement is built in different ways.

The ecological dimension once again separates the cases. A contextualizes the content through situations close to the group's environment. B maintains a clear curricular alignment, although less developed in terms of contextualization. G presents a more visible connection between content, applied situation and educational level.

5.2. Knowledge and skills mobilized in unequal conditions

The comparison between the three cases allows us to argue that didactic-mathematical knowledge and skills are not visible only in the didactic sequence. Rather, they are recognized in the relationship between what the teacher plans, what actually happens in class, the reading they make of that experience, and the adjustments they propose later, in a logic close to the situated reflective practice described for continuing mathematical education in Panama (Morales-Maure & García-Marimón, 2023).

The three cases show this process, although with different profiles. In A, the mobilization of competencies is based above all on interaction, on the contextualization of the content and on a favorable disposition towards working with the group, but under a more restricted mediation. In B, the profile is more balanced: there is a more stable articulation between content,

representation, interaction and use of visual supports. In G, technological mediation acquires more weight and modifies more clearly the way of organizing participation and the treatment of content.

These differences suggest that the conditions of access and use of resources do not operate only on the material plane. They also have an impact on the way in which certain competencies can be deployed and made visible. In some cases, didactic work is based more on interaction and contextualization, in others, in the cognitive organization of the sequence or in technological mediation. For this reason, the inequality of conditions should not be read only as a difference in infrastructure. It also affects the way in which mathematical and didactic activity is organized in the classroom.

Nevertheless, the three cases show that the training received at EMAS is not exhausted in the initial design of a class. What appears most clearly is the ability to review practice based on evidence and to formulate subsequent adjustments. This review is not expressed with the same density in all dimensions, but it does allow us to recognize the competence of analysis and didactic intervention in development.

5.3. Scope and limits of technological mediation in the three cases

The three cases allow us to observe that blended mediation does not manifest uniformly across classrooms. In case A, the blended component is less visible: classroom development relies primarily on oral explanation, the board, and direct teacher-student interaction, with minimal integration of virtual resources. In case B, digital tools such as supporting videos and guides function as bridging elements between the virtual platform and face-to-face instruction, facilitating understanding and representation of content without fundamentally altering the overall class structure. In case G, digital resources are fully integrated into instructional planning and enactment, shaping participation, feedback, and real-time adjustments, and becoming an essential component of how the class is conducted.

These contrasts highlight that the scope of blended mediation is not determined merely by the quantity of resources available, but by how these resources are mobilized within the didactic activity. In A, limitations in access restrict the diversity of support but do not prevent mathematical work, reflection, or student engagement. In B, favorable infrastructure allows for richer representations, guided practice, and collaborative exchanges. In G, technology is intertwined with instructional decisions, contributing directly to task sequencing, student participation, and assessment. This shows that digital tools mediate learning only in relation to pedagogical design, teacher decisions, and classroom interaction, rather than acting as autonomous determinants of quality.

At the same time, these cases show that technological mediation alone does not explain the learning outcomes or instructional effectiveness. Even with restricted digital participation, A sustains interaction, contextualization, and reflective practice. B maintains coherent articulation between content, representation, and peer exchange. G demonstrates the clearest integration of blended learning, yet this integration is meaningful because it is embedded in concrete didactic choices. Availability of resources creates possibilities but does not replace the teacher's role in guiding content, interaction, and instructional sequencing.

Therefore, the main limitation of focusing solely on technology would be to reduce differences between cases to a question of infrastructure. The evidence reveals a complex interplay between digital conditions, pedagogical design, participation structures, and classroom outcomes. In this sense, blended mediation should be understood as a dimension of teaching work—shaped by design, enactment, and feedback cycles across face-to-face and virtual modalities—rather than as an isolated indicator of quality.

6. Conclusions

This study examined how secondary school teachers participating in a higher education professional development program participating in a higher education professional development program participating in a higher education professional development program participating in a higher education professional development program participating in the EMAS Diploma Program, conceived as a blended professional development environment, mobilized didactic-mathematical knowledge and competencies when designing, implementing, assessing, and redesigning mathematical instruction under unequal conditions of digital access and participation. The analysis of cases A, B, and G showed that this process cannot be understood only from what teachers planned, but from the relationship among the didactic sequence, classroom enactment, the evidence produced, teacher reflection, and redesign across face-to-face and virtual conditions. The findings reveal a shared professional basis across the three cases, but also important differences in how the blended model was enacted in practice. In Case A, the blended component was less visible in classroom enactment, and teaching relied more strongly on direct interaction and contextualized explanation in face-to-face settings. In Case B, a more balanced profile was observed, with a clearer articulation between content, representation, interaction, and digital support resources that helped connect the virtual and in-person components of instruction. In Case G, blended enactment was more visible, as digital tools played a stronger role in organizing participation, supporting feedback, and shaping the treatment of mathematical content. These differences suggest that digital access and participation do not operate only at a material level but also shape how teacher competencies are enacted and made visible within blended learning environments.

The study also shows that a shared blended training framework does not produce identical trajectories of professional action. In all three cases, didactic-mathematical knowledge and competencies were present, but their expression depended on how teachers were able to sustain continuity between the virtual and face-to-face components of teaching in their particular school contexts. For this reason, the contribution of the study is not to establish hierarchies among cases, but to show that the same blended professional development process can lead to different forms of instructional enactment when it is translated into non-equivalent educational realities.

At the same time, the findings indicate that blended mediation should not be interpreted as an isolated indicator of quality. The cases show that the availability of digital resources may expand possibilities for representation, participation, and feedback, but it does not replace the teacher's role in organizing content, sustaining interaction, and reviewing practice. What matters most is not the mere presence of digital tools, but the extent to which teachers are able to integrate face-to-face and virtual resources into coherent instructional pathways that respond to contextual constraints and learning needs.

This study highlights the importance of examining teaching through complete cycles of design, implementation, evidence, reflection, and redesign within blended learning environments. It also suggests that blended teacher education programs should take unequal conditions of digital participation more explicitly into account, preparing teachers not only to design mathematically sound tasks, but also to adapt, connect, and rethink them across face-to-face and virtual settings. In this sense, blended teacher education programs should not only provide digital resources but also support teachers in designing coherent instructional pathways across face-to-face and virtual environments under unequal conditions of participation. From this perspective, further research is needed to deepen the analysis of how blended learning design, institutional conditions, and teacher professional development interact in mathematics education.

Declarations

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Ethics Statement

This study was conducted in accordance with recognized ethical standards for educational research. Informed consent was obtained from all participants, and data were collected, analyzed, and reported with full respect for confidentiality, anonymity, and the protection of participants' identities. The study protocol was reviewed and approved by the Bioethics Committee of the Universidad de Santander, Approval No. CBI-USantander-M-087-2025.

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Conflicts of Interest Statement

The authors declare no conflicts of interest related to the research, authorship, or publication of this article.

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