

Mathematical Modeling of Direct Current Electric Motors in the Adaptation of Vehicles in the Automotive Fleet using Retrofit Technology

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ABSTRACT:

The main motivation of this study is to contribute to the reduction of greenhouse gas emissions caused by internal combustion engines, which are the propulsion elements of vehicles in the automotive fleet of major metropolitan cities. The technical proposal is the use of direct current electric motors. We focus on the mathematical modeling of these electric motors for use in smaller vehicles, with a maximum capacity of 4 passengers, using retrofit technology or by assembling them on new chassis. The results obtained establish that the parameters of these motors satisfy the requirements of a passenger transport vehicle such as a sedan or coupe.

Keywords: Mathematical Model – Temperature – Dc Motor – Gas Emissions – Greenhouse Effect – Electromobility

1) Introduction:

A few decades ago, humanity's concern was to mitigate the greenhouse effect, and the only possibility was to reduce the emission of greenhouse gases such as carbon dioxide. CO₂, For methane CH₄, one approach is the use of electric motors in passenger transport, and one of the alternatives is the use of direct current electric

motors, due to their simplicity in speed control and also in manufacturing.

The proposal to use electric motors in passenger transport vehicles is feasible and easy to implement. [1],

Retrofit technology consists of replacing a vehicle's internal combustion engine with a 100% electric motor. For this reason, the purpose of this study is to establish a mathematical model of this rotating machine and, by simulating it using

Matlab or Simulink, we can observe the output parameters such as armature current, torque, and angular velocity.

2) Methods and Methodology:

The methodology used is applicable within the engineering sciences, which has been based on research (1) the simulation of the operation of a direct current (DC) electric motor for use in small vehicles in order to establish the feasibility of proper operation and energy efficiency. This allows us to conclude that the use of DC electric motors, considering the temperature variation factor, requires the design of a controlled cooling system for the proper functioning of this machine, which is so important for mitigating the greenhouse effect caused by the emission of polluting gases. [1]

A DC electric motor is mainly composed of the rotor and the stator. The armature houses the wire coils mounted on a magnetic core. The operating principle is based on the interaction of the magnetic fields of the rotor and stator, generating a torque that rotates the motor shaft. Electrical power is supplied by a constant voltage DC source, such as a battery. The physical connection between the voltage source and the coils is made with carbon brushes via commutator segments or copper laminations on the rotor. Independent excitation allows for the generation of a constant magnetic field in the stator.

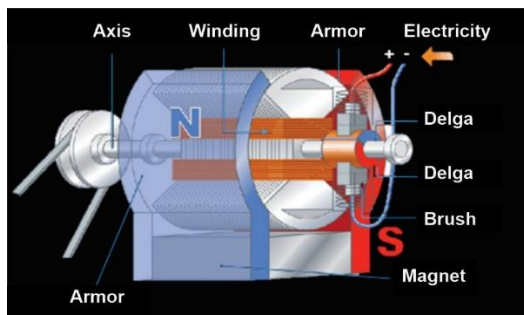


Figure 1

Main components of a DC motor

The mathematical model of a DC motor is formulated as follows:

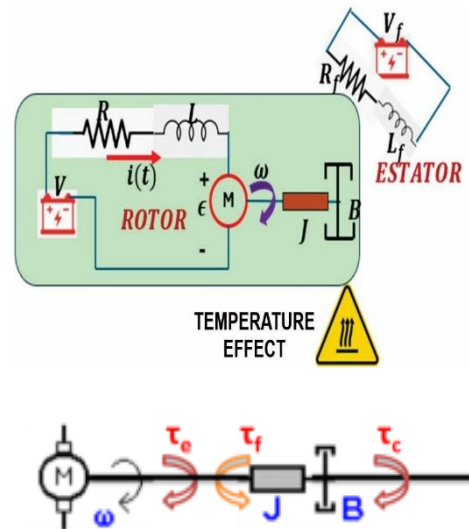


Figure 2

Mathematical model of the DC motor

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Where:

V_i	electromotive force, the source of constant armature voltage (volts)
I	current intensity that flows in the armature coils (amperes)
R_i	electrical resistance of the rotor coil (ohms Ω)
L_i	inductance of the rotor coil (Henries H)
ϵ	counter-electromotive force of the rotor (Volts)
ω	Angular speed of rotor rotation in radians/second ($\frac{rad}{s}$)
α	Angular acceleration of the rotor in radians/ $seconds^2$ ($\frac{rad}{s^2}$)
J	Moment of inertia including the load ($\frac{kg-m^2}{s^2}$)
τ_e	Torque generated by the rotor's magnetic field in newton-meters (N-m)

τ_f	Torque generated by the stator magnetic field in newton-meters (N-m)
τ_c	Resulting torque: $\tau_c = \tau_e - \tau_f$
B	Viscosity coefficient
T	Temperature (°C)
K	Temperature variation constant ($\frac{1}{^\circ\text{C}}$)
U	Coefficient of variation of electrical resistance (linear model)

Note that the following differential equations and relationships between the parameters and variables of the model shown in Figure 2 are fulfilled (Several of these equations were taken from (2) except those involving temperature, which is a contribution from the authors). We apply Kirchhoff's laws to the equivalent circuit of the rotor:

Equation 1

$$\epsilon = k_b \times \omega(t)$$

Where k_b is the counter-electromotive force constant

Equation 2

$$\frac{di}{dt} = \frac{1}{L} [V - i \times R - k_b \times \omega]$$

Equation 3

$$\alpha = \frac{d}{dt} \omega(t)$$

Equation 4

$$\tau_c = J \times \alpha = J \times \frac{d}{dt} \omega(t)$$

Equation 5

$$\tau_f = \beta \times \omega(t)$$

Equation 6

$$\sum \tau = J \times \alpha$$

$$\tau_e - \tau_f = \tau_c$$

Replacing:

Equation 7

$$K_p \times I_i(t) - B \times \omega(t) = J \times \frac{d\omega(t)}{dt}$$

$$K_p \times I_i(t) = J \times \frac{d\omega(t)}{dt} + B \times \omega(t)$$

Solving for $I_i(t)$ in equation (7) and then differentiating with respect to time, we get:

Equation 8

$$I_i(t) = \frac{J \times \frac{d\omega(t)}{dt} + B \times \omega(t)}{K_p}$$

Equation 9

$$\frac{dI_i(t)}{dt} = \frac{J \times \frac{d^2\omega(t)}{dt^2} + B \times \frac{d\omega(t)}{dt}}{K_p}$$

Substituting equations (8) and (9) into equation (2), we will obtain the second-order non-homogeneous linear differential equation with constant coefficients (3).

Equation 10

$$V_i - K_b \times \omega(t) = R_i \times \frac{J \frac{d\omega(t)}{dt} + B\omega(t)}{K_p} + L_i \times \frac{J \frac{d^2\omega(t)}{dt^2} + B \frac{d\omega(t)}{dt}}{K_p}$$

For DC motors with stator-independent excitation, $L_i = 0$ is considered, obtaining:

Equation 11

$$V_i - K_b \times \omega(t) = R_i \times \frac{J \frac{d\omega(t)}{dt} + B\omega(t)}{K_p}$$

For this model, the initial condition is that the rotation speed is 0.

$$t = 0 \rightarrow \omega(0) = 0$$

Then:

Equation 12

$$V_i = \left(\frac{R_i \times J}{K_p} \right) \times \frac{d\omega(t)}{dt} + \left(\frac{R_i \times B}{K_p} + K_b \right) \times \omega(t)$$

We took the transformed version from La Place, resulting in:

Equation 13

$$\begin{aligned} \mathcal{L}[V_i] &= \mathcal{L} \left[\gamma \times \frac{d\omega(t)}{dt} + \beta \times \omega(t) \right], \gamma \\ &= \left(\frac{R_i \times J}{K_p} \right), \beta \\ &= \left(\frac{R_i \times B}{K_p} + K_b \right) \end{aligned}$$

$$\frac{V_i}{s} = \gamma \times s \times \omega(s) + \beta \times \omega(s)$$

$$\frac{V_i}{s} = \omega(s) \times (\gamma s + \beta)$$

Thus the equation becomes:

Equation 14

$$\omega(s) = V_i \times \frac{1}{s} - V_i \times \frac{\frac{1}{\beta}}{\left(s + \frac{\beta}{\gamma} \right)}$$

Taking the inverse transform, the angular velocity as a function of time is obtained.

Equation 15

$$\omega(t) = \frac{V_i}{\beta} \times \left(1 - e^{-\frac{\beta}{\gamma} t} \right)$$

Equation 16

$$T = 25 + Kt$$

Equation 17

$$R = 0.01(1 + UT)$$

This mathematical model allows us to perform several simulations using the Simulink tool; the input parameter is the continuous voltage, for which the input signal is a ramp function.

3] Results: Present the findings of the research paper in this section.

The results of the investigation are shown in the graphs:

1. In Matlab (2023a) the initial parameters of the motor were used according to a motor with a nominal power of 35 kW, as shown below:

%%% Motor Parameters

$V_i = 240$; % [V]

$R_0 = 0.6$; % [ohm] Base resistance at initial T

$L_i = 0.0012$; % [H]

$k_b = 1.8$; % [V·s/rad]

$k_p = 0.4$; % [N·m/A]

$J = 1.0$; % [kg·m²]

$B = 0.2287$; % [N·m·s]

$T_L = 0$; % [N·m] Load torque

$\alpha = 0.019444$; % [°C/s]

$k_R = 3.8e-3$; % [1/°C]

% 3-Stage Starter (adjust as desired)

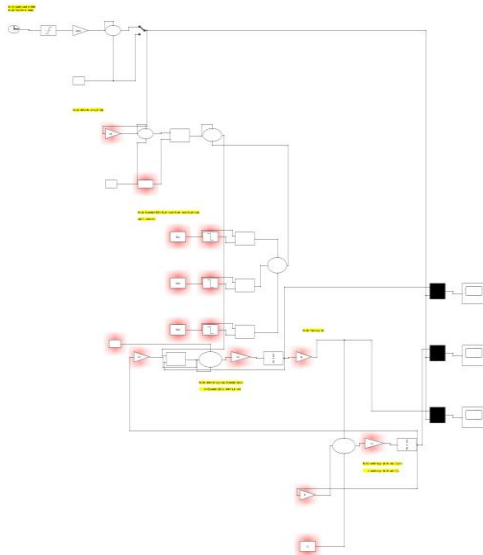
$R_{st1} = 0.8$; % [ohm] Stage 1

$R_{st2} = 0.6$; % [ohm] Stage 2

$R_{st3} = 0.4$; % [ohm] stage 3

$t_1 = 1.5;$ %[s] remove Rst1
 $t_2 = 3;$ %[s] remove Rst2
 $t_3 = 4.5;$ %[s] remove Rst3

The Simulink schematic is shown, a fundamental tool for performing the simulation.



2. Graphs of only the variables (DC motor with armature resistance with variable temperature)

a. Current vs time

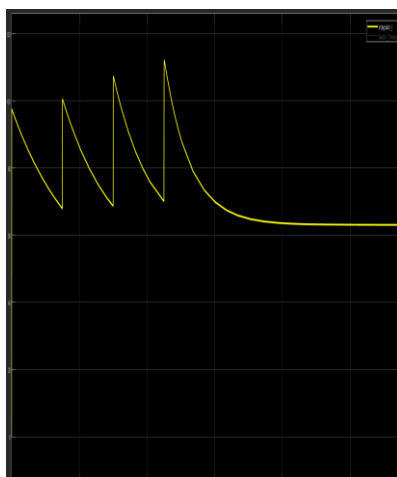


Figure 3 Current vs time

It can stabilize after 1800 seconds (30 minutes), which indicates that it is dangerous for the lifespan of the DC motor. (4)

b. Torque vs time

Value to $t = 10s$

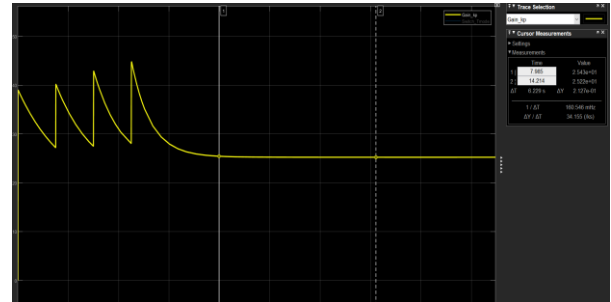


Figure 4 Torque vs time

At 10 seconds the resulting torque against a constant load stabilizes after the transient; it is noticeable that in the transient at startup there is a lot of fluctuation in the resulting torque.

c. Angular velocity – versus time



Figure 5 Angular velocity – versus time

The rotation speed stabilizes after starting; it is observed that the temperature affects the motor parameters.

2.- The simulation results also allow us to compare the two curves, both without considering the temperature and considering the effect of temperature.

a.- Armature current versus time

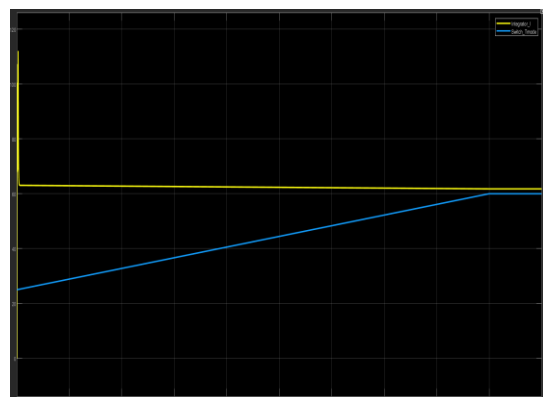


Figure 6 Armature current versus time

It is observed that temperature has an effect that alters the armature current.

b. Torque versus time

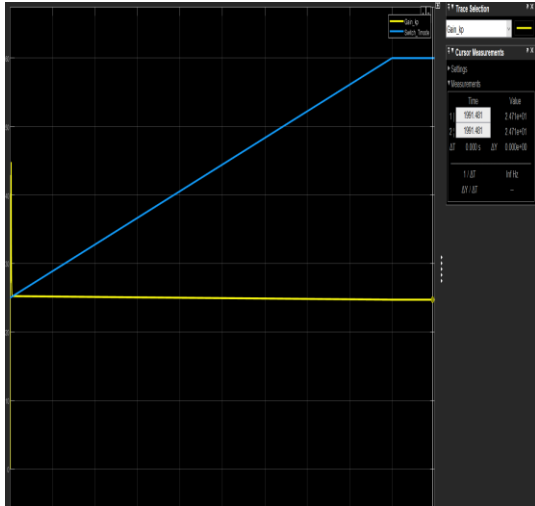


Figure 7 Torque versus time

Due to the temperature, the resulting torque is greater, meaning the motor is required more to meet the load. Therefore, if the temperature is not controlled, it is detrimental to the lifespan of this rotating machine, and its effect on electromobility is that it does not have a fast response time when the vehicle enters inclines or the load per passenger increases.

c.- Angular velocity versus time

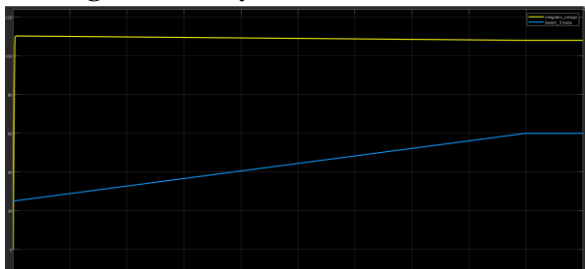


Figure 8 angular velocity versus time

In the case of rotational speed, it decreases due to the effect of temperature.

Research has been conducted on vehicle battery recharging systems in electromobility. Therefore, photovoltaic recharging systems are proposed so that electromobility can fulfill its role of being environmentally friendly.



Figure 9 Battery recharging with solar panels and electronic controller

It is necessary to establish the proper use of electric motors; the power output must be carefully selected for each type of vehicle, as shown in the following figures:



Figure 10 Chassis of an electric vehicle, which houses the motor, its batteries, and control system.

4] Discussion - Provide detailed interpretations and implications of the results.

4a.- The use of electromobility in general requires

considering the effect of temperature in the designs for better motor performance and, of course, the smooth operation of 100% electric vehicles.

4b.- Temperature changes affect the efficiency of electric motors, but from the perspective of greenhouse gas emissions, primarily carbon dioxide (CO₂), any electric machine seemingly reduces emissions. However, the energy source is not considered. Lithium batteries, when recharged, if they do not use renewable energy, can be more harmful than recharging them with an electrical system whose energy matrix is made up of power plants that emit greenhouse gases.

4c.- Electromobility can use direct current electric motors, but it has a limitation for vehicles with more than 4 passengers; for those vehicles, alternating current motors must be used.

4d.- It is debatable whether the reduction of the greenhouse effect produced by the emission gases of internal combustion engine vehicles when they are transformed into hybrid or 100% electric vehicles is effective, because the benefits of electric propulsion machines are only effective if the recharging of the batteries or battery bank is done using exclusively renewable energy sources, such as photovoltaic, wind, geothermal, hydroelectric energy, among others.

4e.- It is recommended to continue investigating the improvement of the efficiency of direct current electric motors as well as three-phase asynchronous electric motors and perhaps even multi-phase ones, which are the future of electromobility.

5] Conclusion – Summarize the main outcomes and their significance.

5a.- When designing vehicles that will use DC motors, it is necessary to consider systems that counteract the negative effect of coil heating and

the influence of the environment in hot areas or summer climates. (6)

5b.- To mitigate the greenhouse effect from the emission of gases in the combustion of gasoline engines, it is necessary to use charging stations with renewable energy, mainly photovoltaic energy from solar panels.

5c.- This research work motivates a new investigation into a technical and viable proposal to reduce the greenhouse effect from the emission of gases in the vehicle fleet of a metropolis.

5d.- In Latin American countries, because their energy matrix consists of only 15% renewable energy, governments must establish a policy to change the energy matrix towards renewable energies such as photovoltaic and wind, and even tidal or geothermal, in order to mitigate the greenhouse effect.

5e.- We finally conclude that the global policy established by the Paris Agreement is viable if governments take climate change seriously, but the only way to achieve this is by implementing programs to convert combustion-powered machines to electrically powered machines with charging in an interconnected electrical system whose energy matrix exceeds 80% renewable energy. Our mission is to continue contributing ideas for adoption by those who make decisions in energy policies.

6] Funding Statement:

This research was conducted at the authors. There is no fund received for this article.

8] Data Availability:

The data has been taken from reference (7).

9] Conflict of interest: The authors declare that there is no conflict of interest.

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